COVARIANT THEORY OF RADIATION DAMPING AND THE SCATTERING OF CHARGED SCALAR MESONS BY NUCLEONS

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ABSTRACT. It is here shown that the strong increase of cross section with the increase of energy which is characteristic feature of all meson theories with derivative couplings and which remains unsolved from the covariant perturbation method of Feynman, can be removed by the inclusion of the influence of radiation damping. In this paper the Heitler's integral equation for the positive meson by proton and that of negative meson by neutron has been solved by the newly formulated variational technique of Goldberger, using the scalar meson field with vector coupling. A comparison of this result with the previous one obtained from the same scattering process using pseudoscalar meson field with pseudoscalar coupling, solved by the semi-variational procedure of Hsueh and Ma, shows that for a proper fit to the experimental result obtained by Steinberger, the value of the coupling constant has to be lowered to one third of the former value, 0.56 used in earlier paper by the author. The superiority of the variational technique of Goldberger to that of Ma and Hseuh has been established by considering the scattering of negative meson by proton. It is found that the exact solution corresponds to the approximate solution of Goldberger.

1. INTRODUCTION

The covariant formalism of Tomonaga and Schwinger (1948) on quantum electrodynamics has been applied successfully by many authors to remove the difficulties of meson theory. By the consistent use of the ideas of charge and mass renormalization, the divergencies arising from higher order radiative corrections have been removed from the covariant S-matrix of Schwinger by some authors They have used the method of calculation devised by Feynmnan (1949) and Dyson (1949). Attempt has also been made to examine whether the cross section for meson scattering could remain finite even at high energy, on account of damping reaction.

This remains unsolved from the perturbation method of Dyson and Feynman unless we consider the Heitler's theory of radiation damping. From the relativistically covariant perturbation methods introduced by Feynman (1949), Ashkin, Simon and Marshak (1950) have calculated the lowest order scattering cross section for meson-nucleon interaction. The important feature to be noted is that the total cross sections decrease with incident meson energy for Ps (Ps) (ie. pseudoscalar meson theory with pseudoscalar coupling) and S(S) (*ie.* scalar meson theory with scalar coupling) in contrast with their increase with energy for the PS (PV) (*ie.* pseudoscalar theory with pseudovector coupling) and S(V) theory (*ie.* scalar meson theory with vector coupling). It is clear that this strong increase of scattering cross section in case of PS(PV) and S(V) theories must be cut down by the damping reaction of the meson field. So we see that Heitler's theory is an effective and preferable alternative to the renormalisation procedure of Schwinger and Tomonaga. But the practical difficulty lies in the fact that the Heitler's integral equation of radiation damping cannot be exactly solved in most cases of the scattering processes. The nature of the difficulties has already been mentioned in an earler paper. The general method is to solve this integral equation by a variational technique. Several authors have suggested different variational procedures of which the variational technique of Hsueh and Ma (1915) is worth mentioning. By this method Basu (1951) and Biswas (1952) have obtained farily good results in the scattering of neutron by proton and the scattering of positive meson by proton respectively.

Recently, a more powerful variational procedure for the approximate solution of the scattering process has been devised by Goldberger (1952). As an example of the use of the variational method he has calculated the scattering process of positive meson by neutron assuming the pseudoscalar meson field using both pseudoscalar and pseudovector coupling. The superiority of this method may be observed from the fact that the result obtained by him is the same as that obtained by Ma and Hsueh (1944) from the exact solution of the integral equation.

In this paper this variational technique has been applied to solve the integral equation for the scattering process of positive meson by proton and that of negative meson by neutron taking into account of the damping reaction using a scalar meson field with vector coupling. The choice of this field is due to two points ; (1) the matrix element for the process is simpler than that in the pseudoscalar theory, (2) the consideration of damping reaction is necessary in this theory with derivative coupling as the cross section increases with increasing energy.

Lastly, a comparison between the theoretical results obtained by the scalar theory and pseudoscalar theory (Biswas, 1952) has been made by means of a graph. The results in scalar theory seem to be too large. The cross section increases ten times in the scalar theory than in the pseudoscalar theory with the same value of the coupling constant which was chosen to be 0.56 for a proper fit to the experimental result of the scattering of π^+ mesons by protons (Steinberger, 1951). Another feature to note is that the value of the coupling constant should be lessened by one third of the above mentioned value in order to be in agreement with the experimental value if we assume scalar meson theory.

In another section it has been shown that this variational technique gives exact solution in case of the scattering of the negative meson by proton with S(V) theory as in PS(PS) theory. The consideration of the damping reaction seems to be unnecessary in PS(PS) theory since the cross section in this case decreases with increasing energy without damping influence. So this damping consideration will yield too low a result for the process. But this is not the case with S(V) theory.

The natural units $\hbar = c = 1$ have been used throughout.

2. COVARIANT FORMALISM

The Schwinger equation in the interaction representation

$$i\frac{\delta}{\delta\sigma(x)}\Psi[\sigma] = H(x)\Psi[\sigma] \quad \ldots \quad (2.1)$$

is solved by the usual invariant perturbation S-matrix which is given by

$$S = 1 + \sum_{n=1}^{\infty} \frac{(-i)^n}{n!} S_n$$

with,

$$S_n = \int_{-\infty}^{\infty} dx' \int_{-\infty}^{\infty} dx'' \ldots\ldots \int_{-\infty}^{\infty} dx^{(n)} P\Big(H(x'),\ H(x''),\ \ldots\ldots\ H(x^{(n)})\Big) \quad \ldots \quad (2.2)$$

where symbol P stands for the Dyson's chronological product. In this form of S-matrix, the damping effect is not exhibited. To do this the usual method is to write the S-matrix in the Cayley form,

$$S = \frac{1 - \frac{i}{2}K}{1 + \frac{i}{2}K} \quad \ldots \quad (2.3)$$

where K is a hermitian matrix.

Writing $$S = 1 - iR \quad \ldots \quad (2.4)$$

where R is the usual reaction matrix (it represents the amplitude of the scattered wave) we obtain from (2.3) and (2.4) the well known Heitler's (1941) and Pauli's (1946) equation of radiation damping, namely,

$$R = K - \frac{i}{2} KR \quad \ldots \quad (2.5)$$

This is the covariant form of Heitler's integral equation. This gives the scattering cross section with damping which is proportional to $|R|^2$.

It has been shown by Schwinger (1948) that the hermitian matrix, K is given by

$$K = \sum_{n=1}^{\infty} K_n$$

with $$K_n = \left(\frac{-i}{2}\right)^{n-1} \int_{-\infty}^{\infty} dx' \ldots \int_{-\infty}^{\infty} dx^{(n)} H(x') \cdot H(x^{(n)}) \epsilon[\sigma', \sigma''] \ldots \epsilon[\sigma^{n-1}, \sigma^n] \quad \ldots \quad (2.8)$$

We can solve the integral equation (2.5) calculating K upto any order required.

Since the total energy-momentum is conserved, the matrix element S, R and K must have the form (Pauli, 1946)

$$(p^1 \ldots p^i \mid A \mid p'_0) = \delta^4(p - p_0)(p' \ldots p^i \mid \bar{A} \mid p'_0 \ldots p^i_0) \quad \ldots \quad (2.9)$$

where $P_0 = \sum_{i=1}^{n} p^i_0$ and $P = \sum_{i=1}^{n} p_i$

are the energy-momentum of four vectors of the initial and final states and $\bar{A}$ is the submatrix of A belonging to the energy shell.

By means of this submatrix, the fundamental equation (2.5) reduces to the form

$$(p \mid R \mid p_0) = (p \mid K \mid p_0) - \frac{i}{2} \int (p \mid K \mid q)((q \mid R \mid p_0) \frac{d^4q}{(2\pi)^3} \delta(q^2 + M^2)\delta^4(Q - p_0) \quad \ldots \quad (I)$$

where $Q = \sum_{i=1}^{n} q_i$

If we denote the initial and final energy-momentum of the nucleon and meson by p_0, k_0 and p, k, respectively, the matrix element K for the meson scattering has the general form (see Fukuda and Miyazima, 1950)

$$(p, k \mid K \mid p_0 k_0) = (2\pi)^4 \varphi_k^{\gamma}(k)\varphi_i(k_0)\{\bar{\psi}(p)\tau_k \tau_i A(p, k, p_0, k_0)\psi(p_0) + \bar{\psi}(p)\tau_i \tau_k B(p, k ; p_0, k_0)\psi(p_0)\} \quad \ldots \quad (2.10)$$

writing

$$(p, k \mid R \mid p_0, k_0) = (2\pi)^4 \varphi_k^{*}(k)\varphi_i(k_0)\{\bar{\psi}(p)\tau_k \tau_i X(p, k ; p_0 k_0)\psi(p_0) + \bar{\psi}(p)\tau_i \tau_k Y(p, k ; p_0, k_0)\psi(p_0)\} \quad \ldots \quad (2.11)$$

and introdicing this in (I) we get a pair of integral equations

$$X(p, k ; p_0, k_0) = A(p, k ; p_0 k_0) - \frac{ip}{(4\pi)^2 w} \int d\Omega' A(p, k ; p_0', k_0')\Lambda(p') X(p', k' ; p_0, k_0) \quad \ldots \quad (2.12)$$

and

$$Y(p, k ; p_0, k_0) = B(p, k ; p_0 k_0) - \frac{ip}{(4\pi)^2 w} \int d\Omega' B(p, k ; p', k')\Lambda(p') Y(p', k' ; p_0, k_0) \quad \ldots \quad (2.13)$$

The $\Lambda(p')$ is the Feynman's projection operator, $\Lambda(p') = \gamma \mathbf{p}' + M$.

The first integral equation is for the process 'negative meson by proton' and the 2nd integral equation is for the process 'positive meson by proton'. Here we deal with the 2nd equation.

3. VARIATIONAL TECHNIQUE OF GOLDBERGER

We now proceed to derive a variational basis of the Heitler's integral equation (I, sec. 2,) according to the newly formulated principle of Goldeberger (1952).

We write the equation (I) in an alternative but more convenient way namely,

$$R_{ba} = G_{ba} + i\pi \sum_c G_{bc} \delta^1(P_c - P_0)\delta(q^2 + M^2) R_{ca} \qquad \ldots \quad (3.1)$$

where P_c and P_0 are energy momentum four vectors of the initial and intermediate states, where

$$P_c = \Sigma q_c \ ; \ P_0 = \Sigma p_0{}'$$

and R_{ba} stands for $(p \mid \bar{R} \mid p_0)$ and G_{ba} for $(p \mid \bar{K} \mid p_0)$

Now defining the matrix multiplication by the following form

$$(RG)_{ba} = \sum_c R_{bc} \delta^1(P_c - P_0)\delta(q^2 + M^2) G_{ca}$$

we proceed to find out the stationary value of the expression L, given by

$$L = (RG)_{ba}(GR)_{ba}[(RR)_{ba} - i\pi(RGR)_{ba}]^{-1}$$

Taking the arbitrary variation δL of L about R we easily see that the stationary value of L is given by

$$\left(\frac{R - G}{i\pi} \right) \qquad (3.3)$$

Goldberger, however, started from the alternative form of the Heitler's integral equation made by Pauli (1949). His starting equation is

$$R_{ba} = G_{ba} + i\pi \sum_e G_{ba} \delta(E - E_e) R_{ea} \ ; \ E - E_0 \qquad \ldots \quad (3.4)$$

The transition probability per unit time from a state characterised by ϕ_a to an initially unoccupied state ϕ_b may be expressed as

$$w_{ba} = (2\pi/\hbar) \mid R_{ba} \mid^2 \delta(E_b - E_a) \qquad \ldots \quad (3.5)$$

Remembering that for a collisional problems, the collision matrix

$$S_{ba} = \delta_{ba} + 2\pi i \delta(E_b - E_a) R_{ba} \qquad \ldots \quad (3.6)$$

is both symmetric and unitary, so that R is symmetric and that G is real and symmetric.

Defining the matrix multiplication by

$$(RG)_{ba} = \sum_c R_{bc} \delta(E - E_c) G_{ca} \qquad (3.7)$$

we see that the stationary value of L, where

$$L = (RG)_{ba}(GR)_{ba}[(RR)_{ba} - i\pi(RGR)_{ba}]^{-1} \qquad \ldots \quad (3.8)$$

about the correct value of R is $(R - G)/i\pi$

The difference between the variational method of Goldberger and that of Ma and Hsueh (1945) may be mentioned. Ma's procedure is based on the fact that the stationary value of M,

$$M = \sum_a \delta(E - E_a)[R^+(R - G - i\pi GR)]_{aa}$$

has been calculated about the correct value of R^+ and not of R. The stationary value obtained is 0. It is also necessary to find out the variation of M about the correct value of R.

We will apply here the variational technique given in (3.2) for the solution of the problem in question.

4. CALCULATION OF MATRIX ELEMENT

We will solve equation (2.5) by calculating K upto 1st. order only, *i.e.*, we will replace K by K_1 in (2.5) and then apply the above variational technique for its solution. It has been shown by Gupta (1951) that upto 1st. order of K, K_1 is the same as S_1 which is the lowest order scattering matrix element in the Feynman's perturbation method of calculation.

We consider here the scattering of scalar charged meson (π^+ or π^-) by neutrons or protons (N or P), schematically shown by

$$\text{I} : \pi^- + P \longrightarrow \pi^- + P \text{ or } \pi^+ + N \longrightarrow \pi^+ + N$$

$$\text{II} : \pi^- + N \longrightarrow \pi^- + N \text{ or } \pi^+ + P \longrightarrow \pi^+ + P$$

The lowest order Feynman diagrams are shown for the above two types of processes (figure 1).

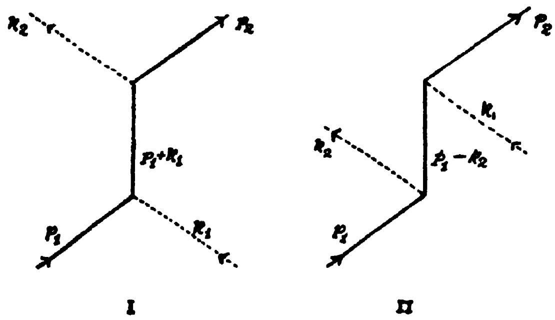

FIG. 1

In process I the incident meson is absorbed by the nucleon before the final meson is emitted since a proton can absorb but cannot emit negative meson and neutron can absorb but not emit positive meson. In process II the emission of the final meson precedes the absorption of the incident meson,

We may now write the covariant matrix element for each process from the corresponding diagram using the scalar meson theory with vector coupling. We should assume the nucleons to obey the Dirac equation. The Feynman's factors corresponding to the martix element (meson energy-momentum four vector $\mathbf{K}$) may be given as follows (see Ashkin, Simon and Marshak, 1950).

	Type of Meson	Type of coupling	Absorption factor for meson K	Emission factor for meson
(1)	Scalar	Vector	$(g/\mu)\mathbf{k}$	$-(g/\mu)\mathbf{k}$
(2)	Propagation factors :		(*i*) $(\mathbf{p}_1+\mathbf{k}_1-M)^{-1}$	for process I
			(*ii*) $(\mathbf{p}_1-\mathbf{k}_2-M)^{-1}$	for process II

(3) At each vertex energy momentum conservation holds.

(4) Each interaction with an external meson, contributes a factor-i.

M denotes the nucleon mass, μ the meson mass. The scalar product of two four vectors will be written in the form

$$\mathbf{A}.\mathbf{B} = A_\mu B_\mu = A^4B^4 - A_1B_1 - A_2B_2 - A_3B_3$$

and each four vector $\mathbf{p}$ is associated with a Dirac operator $\mathbf{p}=p_\mu\gamma_\mu$ where γ_μ are the four matrices $\beta\alpha$, β. The product of two operators A and B satisfies the relation $\mathbf{A}.\mathbf{B}+\mathbf{B}.\mathbf{A}=2A.B$. A nucleon of energy-mementum $\mathbf{p}$ is characterised by a Dirac spinor, u obeying the Dirac equation

$$\mathbf{p}u=Mu \text{ or } \mathbf{u}\mathbf{p}=M\mathbf{u} \text{ where } \mathbf{u}=u^*\beta$$

(u^* is the hermitian conjugate of u). The γ_μ differs from Pauli's by a factor i, for $\mu=1, 2, 3$. And the γ_μ satisfies

$$\gamma_\mu\gamma_\nu+\gamma_\nu\gamma_\mu=2\delta_{\mu\nu} \text{ where } \delta_{44}=+1 \text{ and } \delta_{11}=\delta_{22}=\delta_{33}=-1$$

Let $\mathbf{p}_1$ and $\mathbf{p}_2$ denote the four vector energy-momentum of initial and final nucleons, and $\mathbf{k}_1$ and $\mathbf{k}_2$ the energy-mementum of initial and final mesons.

We have $\mathbf{p}_1+\mathbf{k}_1=\mathbf{p}_2+\mathbf{k}_2$ by energy-momentum conservation and $\mathbf{p}_1^2=\mathbf{p}_2^2=M^2$; $\mathbf{k}_1^2=\mathbf{k}_2^2=\mu^2$. According to the Feynman's rules the matrix elements are given by S(V) theory

$$g^2M_1=-i(g^2/\mu_2)\,\mathbf{u}_2(-\mathbf{k}_2)(\mathbf{p}_1+\mathbf{k}_1-M)^{-1}\mathbf{k}_1\mathbf{n}_1 \text{ for process I} \quad \ldots \quad 4.1$$

$$g^2M_1=-i(g^2/\mu^2)\,\mathbf{u}_2(\mathbf{k}_1)(\mathbf{p}_1-\mathbf{k}_2-M)^{-1}(-\mathbf{k}_2)u_1 \text{ for process II} \quad \ldots \quad 4.2$$

These reduce after a short calculation (Ashkin, Simon, Marshak 1950) to

$$\text{for process I, } i\,(g^2/\mu^2)\mathbf{u}_2(M-W\gamma_1)u_1 \quad \ldots \quad 4.3$$

$$\text{for process II, } -i\,(g^2/\mu^2)\mathbf{u}_2(\gamma_4\epsilon-\gamma p_2)u_1 \quad \ldots \quad 4.4$$

where ϵ is the meson incident energy, p_2 the nucleon final momentum and W is the total energy of the system.

5. SOLUTION $(\pi^+ + P \longrightarrow \pi^{+\prime} + P')$

The relevant matrix element for the process II,

$$(p_2, k_2 \mid G \mid p_1, k_1) = -(ig^2/\mu^2)u_2(\gamma_1\epsilon-\gamma p_2)u_1$$

and in our reference system

$$p_1+k_1=p_2+k_2=0,\ \epsilon=\sqrt{q^2+\mu^2}\ ;\ q=|p_1|=|p_2|=|k|=|k_2|$$

Writing
$$\left.\begin{array}{l}(p_2, k_2 \mid R \mid p_1, k_1)=\mathbf{u}_2\mathbf{R}u_1 \\ (p_2, k_2 \mid G \mid p_1, k_1)=\mathbf{u}_2\mathbf{G}u_1\end{array}\right\} \quad \ldots \quad 5.1$$

We easily obtain the Heitler's integral equation as follows

$$\mathbf{R}(p_2, k_2\,;\,p_1, k_1)=(\mathbf{G}p_2, k_2\,;\,p_1, k_1)+\frac{iq}{32\pi^2W}\int d\Omega'\mathbf{G}(p_2, k_2\,;\,p', k')$$
$$(\gamma_\mu p_\mu+M)\mathbf{R}(p', k'\,;\,p_1, k_1) \quad \ldots \quad 5.2$$

we now turn to the variational problem to solve the equation (4.5) Equation (3.1) has been used in the normal form as it stands.

Writing down the stationary value we get

$$I=\frac{\mathbf{R}-\mathbf{G}}{i\pi}=\left(\frac{q}{32\pi^2w}\right)\times$$
$$\frac{(\int d\Omega'\mathbf{G}(\gamma_\mu p_\mu'+M)\mathbf{R})(\int d\Omega'\mathbf{G}(\gamma_\mu p_\mu'+M)\mathbf{R})}{\int d\Omega'\mathbf{R}(\gamma_\mu p_\mu+M)\mathbf{R}-\left(\frac{iq}{32\pi^2W}\right)\int d\Omega'\int d\Omega''\mathbf{R}(\gamma_\mu p_\mu'+M)\mathbf{G}(\gamma_\mu p_\mu''+M)\mathbf{R}} \quad \ldots \quad 5.3$$

we now take $\mathbf{R}=\mathbf{G}$, as a trial function and we easily get

$$\mathbf{R}=\mathbf{G}+\frac{(iq/32\pi^2 W)(\int d\Omega'\mathbf{G}(\gamma_\mu p_\mu'+M)(\int d\Omega'\mathbf{G}(\gamma_\mu p_\mu'+M)\mathbf{G})}{\int d\Omega'\mathbf{G}(\gamma_\mu p_\mu'+M)\mathbf{G}-(iq/32\pi^2 W)\int d\Omega'\int d\Omega''\mathbf{G}(\gamma_\mu p_\mu'+M)\mathbf{G}(\gamma_\mu p_\mu''+M)\mathbf{G}} \quad \cdots \quad 5{\cdot}3$$

Putting $\mathbf{G}=(-ig^2/\mu^2)(\gamma_{1}\epsilon-\gamma p)$ (we will replace hereafter p_1 and p_2 by p_0 and p denoting initial and final mementa) in the right hand side of the above integrals in (5.3), we perform the integrations which are simple but laborious.

After a straight foward calculation we get for $\mathbf{R}$ the following

$$\mathbf{R}=\left\{\frac{S_1+S_2\gamma_1+S_3(\gamma p_0)+S_4\gamma_1(\gamma p_0)+D(\gamma p)}{T_1^2-T_2^2+T_3^2p^2-T_4^2p^2}\right\} \quad \cdots \quad 5{\cdot}4$$

where S_1, S_2, S_3, S_4, D, T_1, T_2, T_3 and T_4 are all functions of x (as given below) which is q/μ where q denotes the absolute value of the momentum and μ is the meson mass.

Hence the martrix element for the scattering process including radiation damping reduces to

$$R=(\mathbf{u}_2\mathbf{R}\mathbf{u}_1) \text{ where } \mathbf{R} \text{ is given in (5.4).}$$

The differential cross section for the scattering process is in this case given by

$$d\sigma=\frac{d\Omega}{W^2}\,\frac{1}{(2\pi)^2}\,\frac{1}{8}\,\text{spur}\,(\Lambda(p)R\Lambda(p_0)\gamma_4R^+\gamma_4) \quad \cdots \quad 5{\cdot}5$$

where $\Lambda(p)$ is the Feynman's projection operator, $\Lambda(p)=(\gamma_\mu p_\mu+M)$ and we have also taken the average about the initial spin and the summation over the final spin of the nucleon. Performing the integration over all angles of scattering we get for the total scattering cross section,

$$\sigma=g^4\,\frac{1}{2\pi W^2}\,\frac{F(x)}{\{D(x)\}^2} \quad \cdots \quad 5{\cdot}6$$

where
$$\begin{aligned}
F(x)&=(x_2^2+\rho^2)\{S_1(x)+S_2(x)-S_3(x)+S_4(x)-S_5(x)\}\\
&\quad+x^2\{\rho\Gamma_1(x)+2x\Gamma_2(x)-2x\Gamma_3(x)\}-2\rho x\Gamma_4(x)-2x^2\Gamma_5(x)\\
D(x)&=\{\omega(x)\rho(1+x^2)+\rho x_1l_1(x_1^2x_2-x_1x^2)\}^2\\
&\quad-\{\omega(x)(x_1^2x_2-x_1x^2)+x_1^2\rho^2l_1(1+x^2)\}^2\\
&\quad+\{x^2\omega(x)(x_1x_2-x^2)+\rho^2x_1^2l_1x^2\}^2\\
&\quad-\{x^2\omega(x).\rho.x_1+\rho x_1l_1(x_1x_2-x^2)\}^2
\end{aligned}$$

with,

$$\begin{aligned}
S_1(x)&=l^2\{T_1(x)K(x)-T_2(x)L(x)+T_3(x)M(x)-T_4(x)N(x)\}^2\\
S_2(x)&=l^2\{T_1(x)L(x)-T_2(x)K(x)-T_3(x)N(x)+T_4(x)M(x)\}^2\\
S_3(x)&=l^2\{T_1(x)M(x)-T_2(x)N(x)-T_3(x)K(x)+T_4(x)L(x)\}^2\\
S_4(x)&=l^2\{T_1(x)N(x)-T_2(x)M(x)-T_4(x)L(x)-T_4(x)K(x)\}^2\\
S_5(x)&=D(x)\\
\Gamma_1(x)&=\{S_1(x)S_2(x)\}^{1/2}\\
\Gamma_2(x)&=\{S_1(x)S_4(x)\}^{1/2}
\end{aligned}$$

$$\Gamma_3(x) = (S_3(x)S_2(x))^{1/2}$$
$$\Gamma_4(x) = (S_1(x)D(x))^{1/2}$$
$$\Gamma_5(x) = (S_3(x)D(x))^{1/2}$$
$$T_1(x) = \omega(x).\rho.(1 + x^2) + \rho x_1 l_1(x_1^2 x_2 - x^2 x_1)$$
$$T_2(x) = x_1 \rho^2 l_1(1 + x^2) + \omega(x)(x_1^2 x_2 - x^2 x_1)$$
$$T_3(x) = x\omega(x)(x_1 x_2 - x^2) + \rho^2 x_1^2 x^2 l_1$$
$$T_4(x) = x\omega(x)\rho x_1 + \rho x_1 l_1(x_1 x_2 - x^2)$$
$$K(x) = \rho^2 x_1^2(1 + 2x^2) + x_1^2 x_2^2 + x^2 / x^2 - 2x_1 x_2)$$
$$L(x) = (1 + x^2).\rho.(x_1^2 x_2 - x^2)$$
$$M(x) = (1 + x^2).\rho.(x^2 - x_1 x_2)$$
$$N(x) = \rho x x_1^3$$
$$l = l_1 = g.^2 x/8(x_1 + x_2)$$
$$M/\mu = \rho \; ; \; \epsilon/\mu = x_1 \, , \; E/\mu = x_2 \; ; \; q/\mu = x$$
$$\omega(x) = (x_1 x_2 l_1 - x^2 l_1 + 1)$$
$$x_1 = (1 + x_2)^{1/2} \; ; \; x_1 = (\rho^2 + x^2)^{1/2}$$

6. THE SCATTERING OF π MESON BY PROTON

In this section we conclude by remarking that the variational technique formulated by Goldberger is superior to that of Ma and Hsueh (1945). It is here shown that the exact solution of the integral equation regarding the scattering process of scalar negative π-meson by proton closely agrees to the solution obtained by the variational techinque of Goldberger.

The matrix element for the process $(\pi^- + p \rightarrow \pi^{-\prime} + p')$ has been given in section 4.

From (4 3), the matrix element $G = i(g^2/\mu^2)\mathbf{u}_2(M - W\gamma_1)u_1$ can be written as $\mathbf{u}_2\mathbf{G}u_1$ with $\mathbf{G} = a_1 - a_2\gamma_4$

where
$$a_1 = i(g^2/\mu^2)M$$
$$a_2 = i(g^2/\mu^2)W$$

The Heitler's integral equation is given by :

$$\mathbf{R}(p, k \; ; p_0 k_0) = \mathbf{G}(p, k \; ; p_0, k_0) + iq/32\pi^2 w) \int d\Omega' \mathbf{G}(p, k \; ; p', k') \Lambda (p') \mathbf{R}(p', k' \; ; p_0, k_0) \quad \ldots \quad 6.1$$

As in this case the **G** is not involved with angles, **R** may be expressd as $\mathbf{R} = x\mathbf{G}$ where x is some constant not depending on angles. Substituting this value of **R** in (6.1) we find the value of

$$x = \frac{1}{1 - i\lambda}$$

where
$$\lambda = (q/8\pi W)\mathbf{G}(E\gamma_1 + M)$$

Hence
$$\mathbf{R} = \frac{\mathbf{G}}{1 - i(q/8\pi w)\mathbf{G}(E\gamma_4 + M)} \quad \ldots \quad 6.2$$

Now from the variational principle of Goldberger we write the stationary expression for **R**, assuming the trial function $\mathbf{R} = \mathbf{G}$

$$\frac{\mathbf{R}-\mathbf{G}}{i\pi}=\Big(\frac{}{32\pi^3 W}$$

$$(\int d\Omega'\mathbf{G}(\gamma_\mu p_\mu'+M)\mathbf{G})(\int d\Omega'\mathbf{G}(\gamma_\mu p_\mu'+M)\mathbf{G}$$

$$\int d\Omega'\mathbf{G}(\gamma_\mu p_\mu'+M)\mathbf{G}-\left(\frac{iq}{82\pi^2 W}\right)\int d\Omega'\int d\Omega''\mathbf{G}(\gamma_\mu p_\mu'+M)\mathbf{G}(\gamma_\mu p_\mu''+M)\mathbf{G}$$

On performing the integrations we may write

$$\mathbf{R}=\mathbf{G}+\frac{(iq/8\pi W)\mathbf{G}(E\gamma_4+M)}{1-(iq/8\pi W)\mathbf{G}(E\gamma_4+M)}$$

$$=\frac{\mathbf{G}}{1-i(q/8\pi W)\mathbf{G}(E\gamma_4+M)},$$

which is the same result as (6.2) obtained by exactly solving (6.2).

Now from $d\sigma=\frac{d\Omega}{W^2}\;\frac{1}{(2\pi)^2}\;\frac{1}{8}$ spur $(\Lambda(p)\mathbf{R}\Lambda(p_0)\gamma_4\mathbf{R}^*\gamma_4)$ we obtain for the total cross section, σ after performing the angular integrations

$$\sigma=\frac{8\pi}{p^2}\left|\frac{k_1^2+k_2^2-(k_2^2-k_1^2)^2}{1+2(k_1^2+k_2^2)+(k_2^2-k_1^2)^2}\right.$$

where $\quad k_1(x)=\frac{ig}{8\pi}\left\{\rho^2-(\rho^2+x^2)^{1/2}[(1+x^2)^{1/2}+(\rho^2+x^2)^{1/2}]x/(x_1+x_1)\right\}$

$$k_2(x)=\frac{ig^2}{8\pi}\left\{\rho(\rho^2+x^2)^{1/2}-\rho[(1+x^2)^{1/2}+(\rho^{22}+x^2)^{1/2}]x/(x_1+x_2)\right\}$$

with ρ, x_1, x_2, x the same as in (5.6).

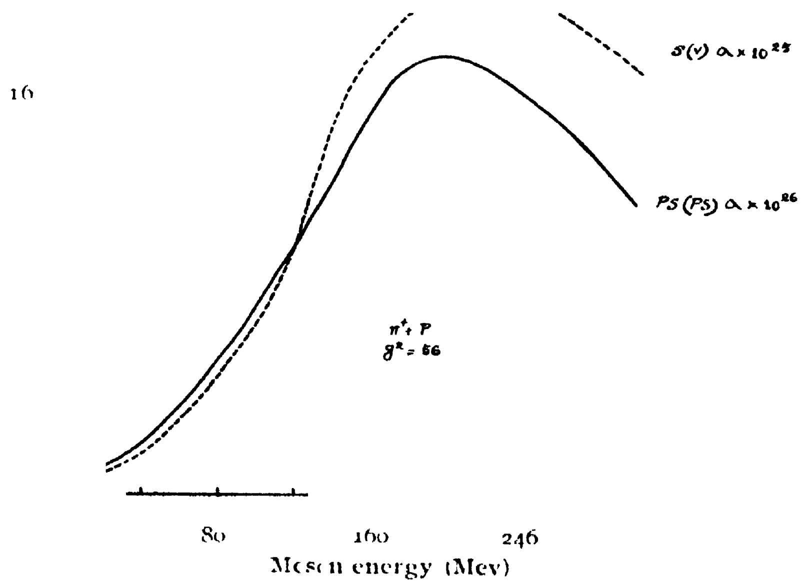

FIG. 2

7. COMPARISON WITH EXPERIMENT

The energy dependence of the total cross section including radiation damping has been shown by means of the accompanying graph (figure 2). The dotted curve drawn is from the theoretical result obtained here using the scalar meson field with vector coupling. The thicker line is from the previous result (Biswas, 1952) for the same process but using pseudoscalar meson field with pseudoscalar coupling. The value of the coupling constant assumed there was 0.56. The same value is also taken for the scalar theory, but with this value the scattering cross section is larger than that in the experimental result of Steinberger (1951). The value is 10 times larger than the former.

The ratio ρ for M/μ is taken to be 6.67 since $\mu = 276 \times m_e$ (m_e = mass of electron). In the scalar meson field the radiation reaction begins to assert itself from 180 Mev. In order to fit the theoretical values in agreement with the experimental one (Steinberger, Sachs and Anderson, 1951) it is necessary to lower down the value of the coupling constant to one third of the value assumed here. So we come to this conclusion that the coupling constant in scalar meson field with vector coupling should be lower than that in the pseudoscalar meson field with pseudoscalar coupling.

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REFERENCES

Ashkin, J. and others, 1950, *Prog. Theor. Phys.*, **5**, 631.
Basu, D., 1951, *Ind J. Phys.*, **25**, 246.
Biswas, S. N., 1952, *Ind J Phys.*, **26**, 61½.
Dyson, F. J., 1949, *Phys. Rev*, **75**, 1736.
Feynman, R. P. 1949, *Phys. Rev.*, **76**, 748, 760
Goldberger, M. L., 1951, *Phys. Rev.*, **84**, 929.
Gupta, S. N., 1951, *Proc. Camb. Phil. Soc.*, **47**, 454
Heilter, W, 1941, *Proc. Camb. Soc.*, **37**, 291.
Hsueh and Ma, 1945, *Phys Rev.*, **67**, 303.
Nobuvki Fukuda and T. Mivazima, 1950, *Prog Theor. Phys*, **5**, 849.
Pauli, W., 1946, Meson Theory of Nuclear forces p 39
Schwinwer, J., 1948, *Phys. Rev.*, **74**, 1439.
Steinberger, 1951, *Phys. Rev*, **82**, 958.